IISER TVM	Varsha 2013
PHY 4105	Problem Set 2 (Due on 3/9/2013)

1 Classical Information Theory

a. Prove the strong subadditivity relation for Shannon entropy

$$H(X,Y,Z) + H(Y) \leq H(X,Y) + H(Y,Z)$$

When does the equality hold?

b. If we roll two six-sided dice and add the numbers on their upper faces, the sum could be any value between 2 and 12. Find the probability for each sum value, and calculate the entropy of the distribution. How does this compare to log 6 (the entropy for one dice) and log 36 (the entropy of the full two-dice configuration, not just the sum)?

c. Suppose that X and X' are independent, identically-distributed random variables. Show that the probability the two agree is at least

$$\Pr(X = X') \geq 2^{-H(X)}$$

d. Suppose that X and Y are two random variables with a joint distribution. For each y, we have the conditional distribution $p(x|y)$ over x. The y's themselves occur with probabilities $p(y)$, leading to an average distribution $p(x)$. Use the concavity of the entropy function to obtain the result that $H(X) \geq H(X|Y)$.

e. Prove that the sum of two or more convex (or concave) functions is also convex (or concave).

2 Basic Tools for Quantum Mechanics

a. Any normal operator M on a vector space V is diagonal with respect to some orthonormal basis for V. Prove this result and its converse.

b. Show that a positive operator is necessarily Hermitian.

c. Show that the eigenvalues of a projector P are all either 0 or 1.

d. Show that a normal matrix is Hermitian if and only if it has real eigenvalues.

e. State and prove the Singular Value Decomposition Theorem in linear algebra.